22/11/24

1. a) $A = \langle 1, 3 \rangle$, $B = \langle 3, 4 \rangle$

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

$$3 + 12 = \sqrt{1^2 + 3^2}\sqrt{3^2 + 4^2}\cos\theta$$

$$\cos\theta = \frac{15}{\sqrt{10}\,3\sqrt{5}}$$

$$= \frac{15}{15\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ$$

(i) $|\vec{A}|\cos\theta = \sqrt{10}\left(\frac{\sqrt{2}}{2}\right)$

$$= \frac{2\sqrt{5}}{2}$$

$$= \sqrt{5}$$

(ii) $|\vec{B}|\cos\theta = 3\sqrt{5}\left(\frac{\sqrt{2}}{2}\right)$

$$= \frac{3}{2}\sqrt{10}$$

b) $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$

$A = \langle 3, 5, 7 \rangle$

$B = \langle 3, 4, 0 \rangle$

$= \dfrac{9+20+0}{\sqrt{3^2+5^2+7^2}\sqrt{3^2+4^2}}$

$= \dfrac{29}{\sqrt{83}\cdot 5}$

$= 0.6366$

$\theta = 50.46^\circ$

$|A|\cos\theta = 5.8$

2. $A = \langle a, 2 \rangle$, $B = \langle 1, 3 \rangle$

$|A| \cos\theta = 0$ only when

$\cos\theta = 0$, $\theta = 90^\circ$

$\vec{A} \cdot \vec{B} = 0$

$a + 6 = 0$

$\therefore a = -6$

$|A| \cos\theta < 0$

$\cos\theta < 0$

$$\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} < 0$$

$\vec{A} \cdot \vec{B} < 0$

$a + 6 < 0$

$a < -6$

3. $\theta = 90^\circ$